1 Operations on Matrices

1.1 Addition and Subtraction

Exercise 1.1

Given: $A = \begin{bmatrix} 7 & -1 \\ 6 & 9 \end{bmatrix}, B = \begin{bmatrix} 0 & 4 \\ 3 & -2 \end{bmatrix}, C = \begin{bmatrix} 8 & 3 \\ 6 & 1 \end{bmatrix}$, find:

1. A + B
2. C − A
3. A − C + B
4. 3 A
5. 2.5 C
6. 2 A − 3 B
7. 4 B + 2 C
8. A + 2 B − 3 C
9. 2 C − 3 A + 2 B

Exercise 1.2

Given: $A = \begin{bmatrix} 2 & 1 & 3 \\ 5 & 0 & 9 \\ 17 & 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 3 & 12 & 1 \\ 2 & 4 & 1 \\ 1 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 8 & 12 \\ 1 & 9 & 20 \\ 1 & 7 & 6 \end{bmatrix}, D = \begin{bmatrix} 5 & 22 & 4 \\ -5 & 9 & -1 \\ 3 & 8 & -7 \end{bmatrix}$, find:

1. A + B
2. A − B
3. B − A
4. C + A
5. C − A
6. 4 C − 3 D
7. 2 B − C
8. D + C
9. 2 C − 3 A + 4 D
10. D − 2 A
11. 2 C + D − 4 A
12. A + B − C − D
13. D − A + C − B

Exercise 1.3

Verify that (A + B) + C = A + (B + C) and (A + B) − C = A + (B − C) for:

1. $A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} -1 & 7 \\ 8 & 4 \end{bmatrix}, C = \begin{bmatrix} 3 & 4 \\ 1 & 9 \end{bmatrix}$
2. $A = \begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix}, B = \begin{bmatrix} 7 & 1 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} 1 & -2 \\ 0 & 2 \end{bmatrix}$

1.2 Multiplication

Exercise 1.4

Given: $A = \begin{bmatrix} 2 & 8 \\ 3 & 0 \\ 5 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 3 & 8 \end{bmatrix}, C = \begin{bmatrix} 7 & 2 \\ 6 & 3 \end{bmatrix}$, find:

1. Is AB defined? Calculate AB. Can you calculate BA? Why?
2. Is BC defined? Calculate BC. Is CB defined? If, so calculate CB. Is it true that BC = CB.

Exercise 1.5

Test the associative law of multiplication with the following matrices:

1. $$\begin{bmatrix} 5 & 3 \\ 0 & 5 \end{bmatrix}, \begin{bmatrix} -8 & 0 & 7 \\ 1 & 3 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 3 \\ 7 & 1 \end{bmatrix}$$
2. $$\begin{bmatrix} 2 & 0 \\ -1 & 4 \end{bmatrix}, \begin{bmatrix} 7 & 0 & 8 \\ -1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 4 \\ 1 & 2 \\ 0 & 0 \end{bmatrix}$$

Exercise 1.6

Find the product matrices in the following (in each case, append beneath every matrix a dimension indicator):

1. $\begin{bmatrix} 7 & -1 \\ 6 & 9 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ 3 & -2 \end{bmatrix}$

2. $\begin{bmatrix} 8 & 3 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} 7 & -1 \\ 6 & 9 \end{bmatrix}$

3. $\begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 4 \\ 2 & 3 & 0 \end{bmatrix}\begin{bmatrix} 8 & 0 \\ 0 & 1 \\ 3 & 5 \end{bmatrix}$

4. $\begin{bmatrix} 3 & 5 & 0 \\ 4 & 2 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

5. $\begin{bmatrix} 6 & 5 & -1 \\ 1 & 0 & 4 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 5 & 2 \\ 0 & 1 \end{bmatrix}$

6. $\begin{bmatrix} a & b & c \end{bmatrix}\begin{bmatrix} 7 & 0 \\ 0 & 2 \\ 1 & 4 \end{bmatrix}$

7. $\begin{bmatrix} 0 & 2 & 4 \\ 3 & 0 & 4 \\ 1 & 3 & 0 \end{bmatrix}\begin{bmatrix} 9 & 0 & 1 \\ 3 & 2 & 1 \\ 1 & 5 & 0 \end{bmatrix}$

8. $\begin{bmatrix} 1 & 2 & 9 \\ 2 & 5 & 0 \\ 3 & 6 & 0 \end{bmatrix}\begin{bmatrix} 9 & 0 & 10 \\ 3 & 0 & 11 \\ 7 & 1 & 0 \end{bmatrix}$

9. $\begin{bmatrix} 5 & 1 & 2 \\ -7 & 2 & 1 \\ 2 & 5 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 & 8 \\ 1 & 8 & 11 \\ 3 & 1 & 0 \end{bmatrix}$

10. $\begin{bmatrix} -1 & 5 & 1 \\ 2 & 5 & 1 \\ 4 & 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 4 \\ 3 & 3 & 1 \\ 3 & 1 & 0 \end{bmatrix}$

11. $\begin{bmatrix} 2 & 1 & 3 \\ 5 & 0 & 9 \\ 17 & 4 & 5 \end{bmatrix}\begin{bmatrix} 3 & 12 & 1 \\ 2 & 4 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

12. $\begin{bmatrix} 1 & 8 & 12 \\ 1 & 9 & 20 \\ 1 & 7 & 6 \end{bmatrix}\begin{bmatrix} 5 & 22 & 4 \\ -5 & 9 & -1 \\ 3 & 8 & -7 \end{bmatrix}$

13. $\begin{bmatrix} 3 & 12 & 1 \\ 2 & 4 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 5 & 22 & 4 \\ -5 & 9 & -1 \\ 3 & 8 & -7 \end{bmatrix}$

14. $\begin{bmatrix} 1 & 8 & 12 \\ 1 & 9 & 20 \\ 1 & 7 & 6 \end{bmatrix}\begin{bmatrix} 2 & 1 & 3 \\ 5 & 0 & 9 \\ 17 & 4 & 5 \end{bmatrix}$

15. $\begin{bmatrix} 3 & 12 & 1 \\ 2 & 4 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 5 & 22 & 4 \\ -5 & 9 & -1 \\ 3 & 8 & -7 \end{bmatrix}$

16. $\begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 4 \\ 2 & 3 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 & 3 \\ 5 & 0 & 9 \\ 17 & 4 & 5 \end{bmatrix}$

17. $\begin{bmatrix} 5 & 22 & 4 \\ -5 & 9 & -1 \\ 3 & 8 & -7 \end{bmatrix}\begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 4 \\ 2 & 3 & 0 \end{bmatrix}$

18. $\begin{bmatrix} 1 & 0 & 2 & 9 \\ 0 & 2 & 1 & 5 \\ 2 & 4 & 1 & 2 \\ 1 & 6 & 3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 & 1 & 0 \\ 4 & 0 & 5 & 3 \\ 7 & 1 & 0 & 4 \\ 3 & 2 & 1 & 1 \end{bmatrix}$

19. $\begin{bmatrix} 2 & 1 & 1 & 9 \\ 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 2 \\ 7 & 0 & 4 & 1 \end{bmatrix}\begin{bmatrix} 9 & 4 & 7 & 0 \\ 4 & 0 & 1 & 0 \\ 6 & 1 & 0 & 4 \\ 3 & 2 & 1 & 0 \end{bmatrix}$

20. $\begin{bmatrix} 8 & 1 & 2 & 0 \\ 3 & 2 & 0 & 0 \\ 2 & 4 & 4 & 1 \\ 5 & 3 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 6 & 2 & 1 \\ 4 & 0 & 0 & 3 \\ 5 & 1 & 0 & 0 \\ 3 & 2 & 3 & 0 \end{bmatrix}$

1.3 Transposition

Exercise 1.7

Find A′ if A is equal to:

1. $\begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix}$

2. $\begin{bmatrix} -1 & 0 \\ 9 & 2 \end{bmatrix}$

3. $\begin{bmatrix} 3 & 7 \\ 3 & -1 \end{bmatrix}$

4. $\begin{bmatrix} 5 & 0 \\ 8 & 1 \end{bmatrix}$

5. $\begin{bmatrix} 6 & 3 \\ 8 & 4 \end{bmatrix}$

6. $\begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$

7. $\begin{bmatrix} 2 & 1 & 3 \\ 4 & -5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

8. $\begin{bmatrix} -7 & 0 & 3 \\ 9 & 1 & 4 \\ 0 & 6 & 5 \end{bmatrix}$

9. $\begin{bmatrix} 2 & 1 & -3 \\ 6 & 3 & 9 \\ 7 & 8 & 9 \end{bmatrix}$

10. $\begin{bmatrix} 8 & 1 & 3 \\ 4 & 0 & 1 \\ 6 & 0 & 3 \end{bmatrix}$

11. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & -7 & 5 \\ 3 & 6 & 9 \end{bmatrix}$

12. $\begin{bmatrix} 4 & 0 & 2 \\ 6 & 0 & 3 \\ 8 & 2 & 3 \end{bmatrix}$

13. $\begin{bmatrix} 1 & 1 & 2 \\ 8 & -1 & 3 \\ 0 & 4 & 3 \end{bmatrix}$

14. $\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$

15. $\begin{bmatrix} x & 5 & 0 \\ 3 & y & 2 \\ 9 & -1 & 8 \end{bmatrix}$

Exercise 1.8

$A = \begin{bmatrix} 0 & 4 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & -8 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 9 \\ 6 & 1 & 1 \end{bmatrix}$. Verify that indeed $(A + B)' = A' + B'$ and $(AC)' = C'A'$.

1.4 Identity Matrix

Exercise 1.9

$A = \begin{bmatrix} -1 & 5 & 7 \\ 0 & -2 & 4 \end{bmatrix}, b' = \begin{bmatrix} 9 & 6 & 0 \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, y = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Indicate dimension of identity matrix and calculate:

1. AI
2. IA
3. Ix
4. bI
5. x′I
6. Iy
7. y′I

2 Solving System of Linear Equations

2.1 Determinant

Exercise 2.1

Use simplified formula and Laplace expansion to find values of determinants of following matrices:

1. $A = \begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix}$
2. $A = \begin{bmatrix} -1 & 0 \\ 9 & 2 \end{bmatrix}$
3. $A = \begin{bmatrix} 3 & 7 \\ 3 & -1 \end{bmatrix}$
4. $A = \begin{bmatrix} 4 & 2 \\ 8 & 0 \end{bmatrix}$
5. $A = \begin{bmatrix} -3 & 0 \\ 2 & 1 \end{bmatrix}$
6. $A = \begin{bmatrix} 2 & 4 \\ 9 & -1 \end{bmatrix}$
7. $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$
8. $A = \begin{bmatrix} -7 & 0 & 3 \\ 9 & 1 & 4 \\ 0 & 6 & 5 \end{bmatrix}$
9. $A = \begin{bmatrix} -2 & 1 & 3 \\ -6 & 3 & 9 \\ 7 & 8 & 9 \end{bmatrix}$
10. $A = \begin{bmatrix} 8 & -1 & 3 \\ 4 & 0 & 1 \\ 6 & 0 & 3 \end{bmatrix}$
11. $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 7 & 5 \\ 3 & 6 & 9 \end{bmatrix}$
12. $A = \begin{bmatrix} 4 & 0 & 2 \\ 6 & 0 & -3 \\ 8 & 2 & 3 \end{bmatrix}$
13. $A = \begin{bmatrix} 1 & 1 & 2 \\ 8 & 11 & 3 \\ 0 & 4 & 3 \end{bmatrix}$
14. $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$
15. $A = \begin{bmatrix} x & 5 & 0 \\ 3 & y & 2 \\ 9 & -1 & 8 \end{bmatrix}$

Exercise 2.2

Evaluate determinants of the following matrices:

1. $\begin{bmatrix} 1 & 2 & 0 & 9 \\ 2 & 3 & 4 & 6 \\ 1 & 6 & 0 & -1 \\ 0 & -5 & 0 & 8 \end{bmatrix}$
2. $\begin{bmatrix} 2 & 7 & 0 & 1 \\ 5 & 6 & 4 & 8 \\ 0 & 0 & 9 & 0 \\ 1 & -3 & 1 & 4 \end{bmatrix}$
3. $\begin{bmatrix} 1 & 3 & 0 & 3 \\ 2 & 1 & 2 & 7 \\ 5 & 1 & 1 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$
4. $\begin{bmatrix} 8 & 0 & 0 & 5 \\ 3 & 0 & 0 & 1 \\ 7 & 1 & 9 & 7 \\ 5 & 0 & 1 & 8 \end{bmatrix}$
5. $\begin{bmatrix} 7 & 0 & 1 & 0 \\ 6 & -9 & 8 & 0 \\ 3 & 8 & 2 & 0 \\ 6 & 3 & 8 & 1 \end{bmatrix}$

Exercise 2.3

Use the determinant $\begin{vmatrix} 4 & 0 & -1 \\ 2 & 1 & -7 \\ 3 & 3 & 9 \end{vmatrix}$ to verify following properties of determinants:

1. $|A| = |A'|$
2. Change of two rows/columns will alter the sign of determinant numerical value
3. The multiplication of one row/column by scalar k will change the value of determinant k-fold.

Exercise 2.4

Which properties of determinants enable us to write the following?

1. $\begin{vmatrix} 9 & 18 \\ 27 & 56 \end{vmatrix} = \begin{vmatrix} 9 & 18 \\ 0 & 2 \end{vmatrix}$
2. $\begin{vmatrix} 9 & 27 \\ 4 & 2 \end{vmatrix} = 18 \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix}$

2.2 Inverse Matrix

Exercise 2.5

Find the inverse of each of the following matrices:

1. $\begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix}$
2. $\begin{bmatrix} -1 & 0 \\ 9 & 2 \end{bmatrix}$
3. $\begin{bmatrix} 3 & 7 \\ 3 & -1 \end{bmatrix}$
4. $\begin{bmatrix} 4 & -2 & 1 \\ 7 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$
5. $\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 3 \\ 4 & 0 & 2 \end{bmatrix}$
6. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$
7. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2.3 Solving System of Linear Equations

Exercise 2.6

Solve the system Ax = d by matrix inversion:

1. $\begin{cases} 4x + 3y = 28 \\ 2x + 5y = 42 \end{cases}$
2. $\begin{cases} 4x_1 + x_2 + -5x_3 = 8 \\ -2x_1 + 3x_2 + x_3 = 12 \\ 3\mathrm{x}_1 - \mathrm{x}_2 + 4\mathrm{x}_3 = 5 \end{cases}$

Exercise 2.7

Use Cramer's rule and matrix inversion to solve the following equation systems:

1. $\begin{cases} 3x_1 - 2x_2 = 6 \\ 2x_1 + x_2 = 11 \end{cases}$
2. $\begin{cases} -x_1 + 3x_2 = -3 \\ 4x_1 - x_2 = 12 \end{cases}$
3. $\begin{cases} 8x_1 - 7x_2 = 9 \\ x_1 + x_2 = 3 \end{cases}$
4. $\begin{cases} 5x_1 + 9x_2 = 14 \\ 7x_1 - 3x_2 = 4 \end{cases}$
5. $\begin{cases} 8x_1 - x_2 = 16 \\ 2x_2 + 5x_3 = 5 \\ 2x_1 + 3x_3 = 7 \end{cases}$
6. $\begin{cases} -x_1 + 3x_2 + 2x_3 = 24 \\ x_1 + x_3 = 6 \\ 5x_2 - x_3 = 8 \end{cases}$
7. $\begin{cases} 4x + 3y - 2z = 1 \\ x + 2y = 6 \\ 3x + z = 4 \end{cases}$
8. $\begin{cases} -x + y + z = a \\ x - y + z = b \\ x + y - z = c \end{cases}$
9. $\begin{cases} x + y + z = 0 \\ 2x - y - z = -3 \\ 4x - 5y - 3z = -7 \end{cases}$
10. $\begin{cases} x - y + 2z = -3 \\ -x + y + z = 0 \\ 2x - y + 2z = -3 \end{cases}$
11. $\begin{cases} 2x + y + z = 0 \\ 4x - 3y + z = 1 \\ 6x + 2z = -2 \end{cases}$
12. $\begin{cases} x + y + z + u = 0 \\ -x + 2y - 2z + 3u = 0 \\ 2x + 3y + 3z + u = 0 \\ 3y - z + 4u = 1 \end{cases}$
13. $\begin{cases} x + y + z + t = -2 \\ -x + y - z - t = 0 \\ x - y - z - t = 1 \\ 2x - y - z - 3t = -1 \end{cases}$

3 Linear Spaces

3.1 Graphical Interpretation of Vectors

Exercise 3.1

Given $u' = \begin{bmatrix} 5 & 1 \end{bmatrix}$ and $v' = \begin{bmatrix} 0 & 3 \end{bmatrix}$, find the following graphically:

1. $2\,v$
2. $u + v$
3. $u - v$
4. $v - u$
5. $2\,u + 3\,v$
6. $4\,u - 2\,v$

Exercise 3.2

Verify whether the following vectors are linearly independent:

1. $\mathbf{a} = [1, 0]$ and $\mathbf{b} = [0, 1]$
2. $\mathbf{a} = [3, 4]$ and $\mathbf{b} = [-3, -4]$
3. $\mathbf{a} = [3, 4]$ and $\mathbf{b} = [4, 3]$
4. $\mathbf{a} = [3, 4]$ and $\mathbf{b} = [1, 0]$
5. $\mathbf{a} = [1, 0, 0]$, $\mathbf{b} = [0, 2, 0]$ and $\mathbf{c} = [0, 0, 8]$
6. $\mathbf{a} = [1, 2, 1]$, $\mathbf{b} = [0, 2, 0]$ and $\mathbf{c} = [-1, 2, 4]$

3.2 Dot Product

Exercise 3.3

Given $u' = \begin{bmatrix} 3 & 4 \end{bmatrix}$ and $v' = \begin{bmatrix} 9 & 7 \end{bmatrix}$ find:

1. $u'v$
2. uv
3. vu
4. vu'

Exercise 3.4

Given $u' = \begin{bmatrix} 5 & 1 & 3 \end{bmatrix}$, $v' = \begin{bmatrix} 3 & 1 & -1 \end{bmatrix}$, $w' = \begin{bmatrix} 7 & 5 & 8 \end{bmatrix}$, and $x' = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$ find:

1. uv'
2. uw'
3. xx'
4. $v'u$
5. $u'v$
6. $w'x$
7. $u'u$
8. $x'x$

Exercise 3.5

Find the cosine of the angle between vectors a and b:

1. $\mathbf{a} = [1, 0]$ and $\mathbf{b} = [0, 1]$
2. $\mathbf{a} = [3, 4]$ and $\mathbf{b} = [-3, -4]$
3. $\mathbf{a} = [3, 4]$ and $\mathbf{b} = [4, 3]$
4. $\mathbf{a} = [3, 4]$ and $\mathbf{b} = [1, 0]$
5. $\mathbf{a} = [1, 2, 3]$ and $\mathbf{b} = [-1, 2, 4]$
6. $\mathbf{a} = [1, 2, 1]$ and $\mathbf{b} = [-1, 2, 4]$

3.3 Quadratic Forms

Exercise 3.6

Express each of the following quadratic forms as a matrix product involving symmetric coefficient matrix:

1. $q = 4x_1^2 - 4x_1x_2 + 9x_2^2$
2. $q = x_1^2 + 7x_1x_2 + 3x_2^2$
3. $q = 8x_1x_2 - x_1^2 + 5x_2^2$
4. $q = 6x_!x_2 + 5x_2^2 - 2x_1^2$
5. $q = 3x_1^2 - 2x_1x_2 + 4x_1x_3 + 5x_2^2 + 4x_3^2 - 2x_2x_3$

3.4 Eigenvalues and Eigenvectors

Exercise 3.7

Find eigenvalues of the following matrices:

a) $A = \begin{bmatrix} -2 & 2 \\ 2 & -4 \end{bmatrix}$ b) $A = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}$

c) $A = \begin{bmatrix} 5 & 3 \\ 3 & 0 \end{bmatrix}$ d) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Exercise 3.8

Determine the eigenvalues, eigenvectors and trace of the matrices below. In each case, check whether the sum of all eigenvalues is equal to the trace.

a) $A = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ c) $A = \begin{bmatrix} 5 & 1 \\ 15 & 3 \end{bmatrix}$

d) $A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$ e) $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ f) $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$

g) $A = \begin{bmatrix} -5 & 0 \\ -7 & 4 \end{bmatrix}$ h) $A = \begin{bmatrix} -5 & 2 \\ 0 & 6 \end{bmatrix}$ i) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

j) $A = \begin{bmatrix} 1 & 4 & 4 \\ 0 & 2 & 0 \\ 0 & -2 & -1 \end{bmatrix}$ k) $A = \begin{bmatrix} 2 & 2 & -2 \\ 1 & 3 & -1 \\ -1 & 1 & 1 \end{bmatrix}$ l) $A = \begin{bmatrix} -1 & 3 & 0 \\ 0 & -2 & 0 \\ 4 & -6 & 1 \end{bmatrix}$

m) $A = \begin{bmatrix} 1 & -2 & -2 \\ -4 & -11 & -8 \\ 4 & 13 & 10 \end{bmatrix}$ n) $A = \begin{bmatrix} 2 & 6 & 6 \\ 0 & 3 & 0 \\ 0 & -3 & -1 \end{bmatrix}$ o) $A = \begin{bmatrix} 3 & 10 & 10 \\ -1 & 0 & -2 \\ 1 & -2 & 0 \end{bmatrix}$

Exercise 3.9

Determine whether the matrices from Exercise 3.6 are definite or indefinite. If they are definite, determine whether they are positive, semi-positive, negative or semi-negative definite.