1 Summation and Product Notation

Exercise 1.1

For the set $\{x_1 = 4,\ x_2 = 3,\ x_3 = 2,\ x_4 = -1,\ x_5 = 10,\ x_6 = 0,\ x_7 = -11\}$ calculate:

1. $\sum_{i=1}^{5} x_i$
2. $\sum_{i=2}^{3} x_i$
3. $\sum_{i=4}^{7} x_i$
4. $\sum_{i=1}^{7} x_i$
5. $\prod_{i=2}^{4} x_i$
6. $\prod_{i=1}^{3} x_i$
7. $\prod_{i=1}^{7} x_i$
8. $\prod_{i=4}^{5} x_i$

Exercise 1.2

Expand the following summation expressions:

1. $\sum\limits_{i=1}^{5} x_i$
2. $\sum\limits_{i=5}^{8} a_i x_i$
3. $\sum\limits_{i=1}^{4} b x_i$
4. $\sum\limits_{i=1}^{n} a_i x^{i-1}$
5. $\sum\limits_{i=0}^{3} (x-i)^2$

Exercise 1.3

Rewrite the following in Σ notation:

1. $x_1(x_1 - 1) + 2x_2(x_2 - 1) + 3x_3(x_3 - 1)$
2. $a_2(x_3 - 2) + a_3(x_4 - 3) + a_4(x_5 - 4)$
3. $\frac{1}{x} + \frac{1}{x^2} + \ldots + \frac{1}{x^n}$
4. $1 + \frac{1}{x} + \frac{1}{x^2} + \ldots + \frac{1}{x^n}$

Exercise 1.4

Show that the following are true:

1. $\left(\sum\limits_{i=0}^{n} x_i\right) + x_{n+1} = \sum\limits_{i=0}^{n+1} x_i$
2. $\sum\limits_{j=1}^{n} a b_j y_j = a \sum\limits_{j=1}^{n} b_j y_j$
3. $\sum\limits_{j=1}^{n} x_j + y_i = \sum\limits_{j=1}^{n} x_j + \sum\limits_{j=1}^{n} y_j$

2 Exponentiation and Logarithms

Exercise 2.1

Simplify:

1. $x^3 \cdot x^7$
2. $x^{-3} \cdot x^5$
3. $x^{-5} \cdot x^4$
4. $x^3 : x^4$
5. $x^3 : x^{-3}$
6. $a^{-7} : a^{-7}$
7. $\left(x^3\right)^5$
8. $\left[\left(c^3\right)^3\right]^3$
9. $\left[\left(a^2\right)^b\right]^2$
10. $\log_2 16$
11. $\log_{\frac{1}{2}} 16$
12. $\log_2 64$
13. $\log_{\frac{1}{2}} 64$
14. $\log_3 \mathrm{x}^{\frac{1}{2}}$
15. $\ln \mathrm{e}$
16. $\ln\left(\mathrm{e}^8\right)$
17. $\ln 2 + \ln 10$
18. $\ln 9 - \ln 3$
19. $\ln 27$
20. $\ln 81$
21. $\ln 2 - \ln 1$
22. $\ln 2 + \ln 15 - \ln 3$

3 Important Functions and Plots

Exercise 3.1

Draw and find X and Y-intercept of the following functions:

1. $y = x$
2. $y = -x$
3. $y = 2x + 3$
4. $y = 3x - 2$
5. $y = -2x + 5$
6. $y = -2x - 1$

Exercise 3.2

Find zeros, coordinates of the vertex and draw the following functions:

1. $y = 3x^2 + x + 5$
2. $y = 16x^2 + 8x + 1$
3. $y = -2x^2 + 3x + 7$
4. $y = -2x^2 + 6x + 5$
5. $y = -2x^2 - 8x + 10$
6. $y = 3x^2 + 2x$
7. $y = -0,5x^2 - 4x - 8$
8. $y = x^2 + 2x - 6$

Exercise 3.3

Draw the following hyperbolic (1 - 7), exponential (8 - 15), logarithmic (16 - 21) and trigonometric (22 - 29) functions:

1. $y = \frac{1}{x}$
2. $y = \frac{1}{x+1} + 3$
3. $y = \frac{1}{x^2} - 2$
4. $y = \frac{2}{(1+x)^2} - 1$
5. $y = \left|\frac{1}{x}\right|$
6. $y = \left|\frac{1}{x^2}\right|$
7. $y = -\left|\frac{1}{x}\right| - 2$
8. $y = 2^x$
9. $y = (\frac{1}{2})^x + 3$
10. $y = 3 \cdot 2^x$
11. $y = 3 \cdot (\frac{1}{2})^x$
12. $y = 2^{2x}$
13. $y = (\frac{1}{2})^{2x}$
14. $y = 2 \cdot 3^{x-1} - 1$
15. $y = 2 \cdot (\frac{1}{4})^{2x-2}$
16. $y = \log_2 x$
17. $y = \log_{\frac{1}{2}} \mathrm{x}$
18. $y = |\log_2 \mathrm{x}|$
19. $y = \left|\log_{\frac{1}{2}} x\right|$
20. $y = \log_2(x-1) + 2$
21. $y = \log_{\frac{1}{2}}(x+1) + 3$
22. $y = \sin(x)$
23. $y = 2\sin(x)$
24. $y = \sin(2x)$
25. $y = \cos(x)$
26. $y = 2\cos(x)$
27. $y = \mathrm{tg}(x)$
28. $y = \mathrm{tg}(2x)$
29. $y = \mathrm{ctg}(x)$

Exercise 3.4

Compare:

1. $y = 2^x$ and $y = \log_2 x$
2. $y = (\frac{1}{2})^x$ and $y = \log_{(\frac{1}{2})} x$

4 Logic

Exercise 4.1

Check whether the following are tautologies:

1. $\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$
2. $[(p \vee q) \wedge \neg p] \rightarrow q$
3. $[(\neg p) \rightarrow q] \leftrightarrow [(\neg q) \rightarrow p]$
4. $[p \rightarrow (\neg p)] \leftrightarrow \neg p$
5. $(p \rightarrow q) \leftrightarrow [(\neg q) \rightarrow \neg p]$
6. $p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$
7. $p \vee (q \wedge r) \leftrightarrow (p \vee q) \wedge (p \vee r)$
8. $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

5 Set Thoery

Exercise 5.1

Given the following: $\Omega = \{0,1,2,3,4,5,6,7,8,9,10\}$, $\mathrm{A} = \{1,2,3,4,5\}$,
$\mathrm{B} = \{0,2,4,6,8,10\}$, $\mathrm{C} = \{2,3,5,7\}$ find:

1. $\mathrm{A} \cup \mathrm{B}$
2. $\mathrm{A}' \cup \mathrm{B}'$
3. $\mathrm{A}' \cap \mathrm{B}$
4. $\mathrm{A} \cap (\mathrm{B} \cup \mathrm{C})$
5. $\mathrm{A} \backslash \mathrm{B}$
6. $\mathrm{A} \backslash (\mathrm{B} \backslash \mathrm{C})$
7. $\mathrm{A} \backslash \mathrm{B} \cap \mathrm{C} \backslash \mathrm{B}$
8. $\mathrm{A} \cap \mathrm{C}$
9. $\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}$
10. $\mathrm{A} \cup (\mathrm{B} \cap \mathrm{C})$
11. $(\mathrm{A} \cup \mathrm{B})'$
12. $\mathrm{B} \backslash \mathrm{A}$
13. $\mathrm{C} \backslash (\mathrm{B} \backslash \mathrm{A})$
14. $(\mathrm{A} \backslash \mathrm{B}) \cap (\mathrm{A} \backslash \mathrm{C})$